18.434 Pset 1 Due 02/18/2020

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Choose 3 problems to write up. I recommend solving even the ones you don't write up. The hints are there in case you want to follow them, but you may solve the problems however you like. As usual, you may collaborate, but the writeup should be in your own words and you should list collaborators and source material. Some of the solutions can be found as proofs in the BH book, but if you use these you'll have to fill in many omitted details.

1. Calculate the spectrum of the n-path P_n. **Hint 1:** It may help to look at the derivation of the spectrum of C_n in the lecture notes for 2/03 (we didn't get to it in class but it's in the notes). **Hint 2:** embed $A(P_n)$ into a larger matrix for which you know the eigenvalues/vectors.

2. Show the formula
$$x^T L(G) x = \sum_{i,j \in E} (x_i - x_j)^2.$$

3. Show that if $v \neq 0$ is in the positive eigenspace of a real symmetric matrix A, i.e. the span of eigenvectors corresponding to positive eigenvalues, then $v^T A v > 0$.

4. Show that the average degree $\bar{d}$ of G is a lower bound for the largest eigenvalue λ_1 of $A(G)$. **Hint:** use Courant-Fischer with an intelligently chosen test vector.

5. Say a nonnegative matrix T is *primitive* if $T^k > 0$ for some integer $k \geq 0$. Show that all the other eigenvalues are strictly smaller in absolute value than the absolute value of the largest positive one.