

Problem set 2

This problem set is due at 11:00 pm on Mar 18, 2021. Instructions same as the first pset; some key points: collaboration is encouraged but you **must** write up your answers in your own words. You are required to list and identify clearly all sources and collaborators except instructors, TA or lecture notes.

To submit your homework, upload it in PDF format using the Gradescope tool in Canvas before the deadline. Unless otherwise indicated, the problems are graded out of 4 points. The graduate questions are worth 2 bonus points each for undergraduates.

1. Exercise 2-2 from the notes on (non-bipartite) matchings.
2. Exercise 2-6 from the notes on (non-bipartite) matchings.
3. (For undergrads; optional for grads) Exercise 2-7, Parts 1 and 2 (Part 3 optional). You may want to look at the new and improved pdf file on non-bipartite matching. **Hint:**¹
4. Consider $S = \{(1, 0, 1), (0, 1, 1), (1, 1, 2), (0, 2, 2)\} \subseteq \mathbb{R}^3$. Describe $\text{lin}(S)$, $\text{aff}(S)$, $\text{cone}(S)$ and $\text{conv}(S)$ (as a polyhedron, in terms of the linear equalities/inequalities). **Hint:** ²
5. Suppose you are given a description of a polyhedron P as the solution set to a system of linear inequalities/equalities. Describe a procedure for finding a description of the conic hull, $\text{cone}(P)$, as the solution set of a system of linear inequalities and equalities. **Hint:** ³
6. For graduate students, exercise 2-5.

¹Try to show that if B is a blossom with respect to a maximum matching in a factor critical graph G , G/B is also factor critical.

² For example, $\text{conv}(\{(0, 1), (1, 0)\}) = \{x : x_1 + x_2 = 1, x_1 \geq 0, x_2 \geq 0\}$. It's ok if you do this in an ad-hoc manner using pictures etc, as long as your final inequalities are correct.

³ You may use that for a polyhedron P , $\text{cone}(P) = \{x : \exists \lambda > 0 \text{ such that } \lambda x \in P\} \cup \{0\}$. Introduce a new variable and use Fourier-Motzkin elimination to get rid of it.