Problem set 3

This problem set is due on Tuesday, April 6, 2021. Instructions same as the first pset; some key points: collaboration is encouraged but you **must** write up your answers in your own words. You are required to list and identify clearly all sources and collaborators except instructors, TA or lecture notes.

To submit your homework, upload it in PDF format using the Gradescope tool in Canvas before the deadline.

1. We will prove strong linear programming duality in a more geometric manner.[1] Consider the linear program of maximizing $c^T x$ over the polyhedron $P = \{x : Ax \leq b\}$ where A has rows $(a_i^T : i = 1, ..., m)$. Assume that the linear program is bounded. We want to show that there is some $y \geq 0$ such that $A^T y = c$ satisfying $b^T y = \max\{c^T x : x \in P\}$.
   (a) Let v_0 be a vertex maximizing $c^T x$. Let I be the set $\{i : a_i^T v_0 = b_i\}$ of constraints that are tight for v_0. Show that there is no vector x such that $a_i^T x \leq 0$ for all $i \in I$ and $c^T x > 0$.
   (b) Conclude that there is some vector $y \geq 0$ such that $y_j = 0$ for $j \notin I$ and and $A^T y = c$. **Hint:** [2]
   (c) Show that this y satisfies $b^T y = c^T v_0$, completing the proof.
2. Exercise 3-10.
3. Exercise 3-12.
4. For graduate students, exercise 3-13.

[1] I consider it more geometric because it uses Farkas lemma straightforwardly, and Farkas lemma is essentially a separating hyperplane theorem (see the latexed notes on polyhedra).

[2] Apply Farkas' lemma.